Graphing Polynomials (Part 1)

These notes are intended as a summary of section 1.4 (p. 37 – 45) in your workbook. You should also read the section for more complete explanations and additional examples.

The Vocabulary of Polynomials

A **term** is the product of one or more numbers and/or variables. A **polynomial** is a combination of one or more terms that are added or subtracted. For example:

$$-5x^2 + 4x - 2$$

In this polynomial expression, there are 3 terms: $-5x^2$, $4x$, and -2. Notice that the **terms are written in descending order of power**.

A term that does not contain a variable is called a **constant**. In terms that have both a number and a variable, the number is called the **coefficient**.

Polynomials are classified by their number of terms. A **monomial** has only one term. A **binomial** has two terms. A **trinomial** has three terms. Expressions with more than 3 terms are simply called **polynomials**.

Properties of Polynomial Functions

In order to more easily graph polynomials, we must be familiar with a number of important properties.

Degree

The **degree** of a polynomial is the highest power of the variable in that polynomial.

Degree	**Name**
0	constant
1	linear
2	quadratic
3	cubic
4	quartic
5	quintic

Example (not in workbook)
State the degree of each of the following polynomials:

a) $f\left(x\right)=2x^3+3x^2-3x+2$

b) $f\left(x\right)=-x^4+3x^3+4x^2-12x-2$

c) $f\left(x\right)=-7x^3+2x^4-8x^2+x^5+12x-1$

d) $f\left(x\right)=x^3-3x^2$

Leading Coefficient

The **leading coefficient** of a polynomial is the coefficient of the term with the highest power.

Example (not in workbook)
Identify the leading coefficient in each of the following polynomials.

a) $f\left(x\right)=2x^3+3x^2-3x+2$

b) $f\left(x\right)=-x^4+3x^3+4x^2-12x-2$

c) $f\left(x\right)=-7x^3+2x^4-8x^2+x^5+12x-1$

d) $f\left(x\right)=x^3-3x^2$

Zeros

The **zeros** of a polynomial are the points where the graph intersects the x-axis. For that reason, they are also known as the x-intercepts. The zeros of a polynomial are found by factoring / division.

Example (not in workbook)
Determine all zeros of $f(x)=2x^4-x^3-14x^2+19x-6$.

Note: The polynomial in this example had 4 zeros. In general, a polynomial will have a number of zeros that is less than or equal to its degree.

Example (not in workbook)
Determine all zeros of $f(x) = x^3 + 2x^2 - 7x + 4$.

Note: This polynomial has 3 zeros. –4 is a zero once, and 1 is a zero twice. How many times a number is a zero of a given polynomial is called its **multiplicity**. So, for this polynomial, –4 is a zero of multiplicity one, and 1 is a zero of multiplicity two.

Y-Intercept

The ***y*-intercept** of a polynomial is the point where its graph intersects the *y*-axis. The *y*-intercept is *always* equal to the constant term of the polynomial. (Note: You can also find the *y*-intercept by setting $x = 0$ and solving for *y*.

Example (not in workbook)
Determine the *y*-intercept of each of the following polynomials.

a) $f(x) = 2x^3 + 3x^2 - 3x + 2$

b) $f(x) = -x^4 + 3x^3 + 4x^2 - 12x - 2$

c) $f(x) = -7x^3 + 2x^4 - 8x^2 + x^5 + 12x - 1$

d) $f(x) = x^3 - 3x^2$

Homework: Supplemental Worksheet #9

Supplemental Worksheet #9

For each of the following polynomials,

- State the degree of the polynomial.
- Identify the leading coefficient.
- Determine all zeros.
- State the multiplicity of each zero.
- Determine the *y*-intercept.

1. $f(x) = x^3 + 3x^2 - 9x + 5$

2. $f(x) = 2x^3 - 9x^2 + 7x + 6$

3. $f(x) = x^5 - 9x^4 + 31x^3 - 51x^2 + 40x - 12$

4. $f(x) = x^3 - 6x^2 + 11x - 6$

5. $f(x) = x^3 + 6x^2 + 3x - 10$

6. $f(x) = x^4 - 8x^2 + 16$